Class : 11

Register Number

COMMOM HALF YEARLY EXAMINATION, 2025 - 26

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I

1. Answer all the questions by choosing the correct answer from the given 4 alternatives $20 \times 1 = 20$
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If $A = \{(x, y) : y = \sin x, x \in R\}$ and $B = \{(x, y) : y = \cos x, x \in R\}$ then $A \cap B$ contains
   (1) cannot be determined (2) infinitely many elements
   (3) only one element (4) no element

2. The range of the function $\frac{1}{1-2\sin x}$ is
   (1) $(-\infty,-1) \cup (\frac{1}{3},\infty)$ (2) $(-\infty,-1] \cup [\frac{1}{3},\infty)$. (3) $[-1,\frac{1}{3}]$ (4) $(-1,\frac{1}{3})$

3. The solution of $5x - 1 < 24$ and $5x + 1 > -24$ is
   (1) $(-5,-4)$ (2) $(-5,4)$ (3) $(-5,5)$ (4) $(4,5)$

4. If 3 is the logarithm of 343 , then the base is
   (1) 7 (2) 5 (3) 9 (4) 6

5. If $\tan 40^\circ = \lambda$, then $\frac{\tan 140^\circ - \tan 130^\circ}{1+\tan 140^\circ \tan 130^\circ} =$
   (1) $\frac{1-\lambda^2}{2\lambda}$ (2) $\frac{1+\lambda^2}{2\lambda}$ (3) $\frac{1+\lambda^2}{\lambda}$ (4) $\frac{1-\lambda^2}{\lambda}$

6. If $\sin\alpha + \cos\alpha = b$, then $\sin 2\alpha$ is equal to
   (1) $b^2 - 1$, if $b > \sqrt{2}$ (2) $b^2 - 1$, if $b \geq 1$ (3) $b^2 - 1$, if $b \geq \sqrt{2}$ (4) $b^2 - 1$, if $b \leq \sqrt{2}$

7. The n^{th} term of the sequence 1,2,4,7,11, ⋯ is
   (1) $\frac{n^2-n+2}{2}$ (2) $\frac{n(n+1)(n+2)}{3}$ (3) $n^3 - 3n^2 + 3n$ (4) $n^3 + 3n^2 + 2n$

8. The coefficient of x^5 in the series e^{-2x} is
   (1) $\frac{3}{2}$ (2) $\frac{2}{3}$ (3) $\frac{4}{15}$ (4) $\frac{-4}{15}$

9. Which of the following point lie on the locus of $3x^2 + 3y^2 - 8x - 12y + 17 = 0$
   (1) $(-2,3)$ (2) $(0,0)$ (3) $(0,-1)$ (4) $(1,2)$

10. The image of the point $(2,3)$ in the line $y = -x$ is
    (1) $(-3,-2)$ (2) $(-3,2)$ (3) $(-2,-3)$ (4) $(3,2)$

11. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, then for what value of λ, $A^2 = 0$?
    (1) 1 (2) 0 (3) ± 1 (4) -1

12. If $A = \begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$, then B is given by

(1) $B = 6A$ (2) $B = -A$ (3) $B = -4A$ (4) $B = 4A$

13. If $\vec{a} + 2\vec{b}$ and $3\vec{a} + m\vec{b}$ are parallel, then the value of m is

(1) 3 (2) 6 (3) $3\frac{1}{3}$ (4) $\frac{1}{6}$

14. If $\vec{a}$ and $\vec{b}$ having same magnitude and angle between them is 60° and their scalar product is $\frac{1}{2}$ then $|\vec{a}|$ is

(1) 1 (2) 2 (3) 3 (4) 7

15. $\lim_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{x}$

(1) $\sqrt{2}$ (2) 0 (3) 1 (4) does not exist

16. If $15C_{3r} = 15C_{r+3}$, then r =

(1) 2 (2) 3 (3) 4 (4) 5

17. The value of $\lim_{x \to k^-} x - \lfloor x \rfloor$, where k is an integer is

(1) 1 (2) −1 (3) 2 (4) 0

18. $\frac{d}{dx}\left(e^{x + 5\log x}\right)$ is

(1) $e^x - \frac{5}{}$ (2) $e^x + \frac{5}{}$ (3) $e^x \cdot x(x+5)$ (4) $e^x \cdot x^4(x+5)$

19. If A and B are any two events, then the probability that exactly one of them occur is

(1) $P(A \cap \bar{B}) + P(\bar{A} \cap B)$ (2) $P(A \cup \bar{B}) + P(\bar{A} \cup B)$

(3) $P(A) + P(B) + 2P(A \cap B)$ (4) $P(A) + P(B) - P(A \cap B)$

20. If A and B are independent events and $P(A) = \frac{3}{5}, P(B) = \frac{1}{5}$ then $P(A \cap B)$

(1) $\frac{1}{25}$ (2) $\frac{3}{25}$ (c) $\frac{1}{5}$ (d) $\frac{4}{25}$

PART – II

1. **Answer any 7 questions** **7 x 2 = 14**
2. **Each question carries 2 marks**
3. **Question number 30 is compulsory**

21. If $n(A) = 10$ and $n(A \cap B) = 3$, find $n((A \cap B)' \cap A)$.

22. Find the value of cos 105°

23. Find the number of ways of arranging the letters of the word BANANA.

24. Write the first 6 terms of the sequences whose n^{th} term a_n is given by $a_n = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$

25. Find the path traced out by the point $\left(ct, \frac{c}{t}\right)$ here $t \neq 0$ is the parameter and c is a constant

26. Find the direction cosines of the line joining (2,3,1) and (3,−1,2)

27. Evaluate the limits: $\lim_{x\to 2} \frac{x^4-16}{x-2}$

28. Evaluate with respect to x : $\int (4x+5)^6 dx$

29. If two coins are tossed simultaneously, then find the probability of getting (i) one head and one tail (ii) atmost two Tails

30. Differentiate $\tan^{-1}(x^2+y^2) = a^2$.

PART – III

1. **Answer any 7 questions** **7 x 3 = 21**
2. **Each question carries 3 marks**
3. **Question number 40 is compulsory**

31. Solve: $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$

32. Find the values of (i) $\sin 18°$ (ii) $\cos 18°$

33. Find the sum of all 4-digit numbers that can be formed using digits 0 , 2 , 5 , 7 , 8 without repetition ?

34. Find the coefficient of x^6 and the coefficient of x^2 in $\left(x^2 - \frac{1}{x^3}\right)^6$.

35. Area of the triangle formed by a line with the coordinate axes, is 36 square units. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of 45° with positive the x-axis.

36. If (k, 2), (2,4) and (3,2) are vertices of the triangle of area 4 square units then determine the value of k.

37. Find $\frac{dy}{dx}$ if $x^4 + x^2y^3 - y^5 = 2x + 1$.

38. Integrate with respect to x : $e^x\left(\frac{x-1}{2x^2}\right)$

39. X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

40. Given $\frac{d}{dx}(F(x)) = \frac{1}{\sqrt{1-(x-1)^2}}$ and $F(1) = 0$, then find $F(x)$.

PART – IV

1. **Answer all the questions** **7 x 5 = 35**
2. **Each question carries 5 marks**

41. a) Let f, g : R → R be defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$. Find f • g

(OR)

b) Integrate with respect to x : $\frac{2x-3}{x^2+4x-12}$

42. a) Simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$.

(OR)

b) Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

43. a) If one root of $k(x-1)^2 = 5x-7$ is double the other root, show that $k = 2$ or -25.

(OR)

b) Evaluate the limits : $\lim_{x\to 0} \frac{\sqrt{x^2+a^2}-a}{\sqrt{x^2+b^2}-b}$

44. a) Solve the equation: $\sin\theta + \sqrt{3}\cos\theta = 1$

(OR)

b) Let A, B, and C be the vertices of a triangle. Let D, E, and F be the midpoints of the sides BC, CA, and AB respectively. Show that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \vec{0}$.

45. a) By the principle of mathematical induction, prove that, for $n \geq 1$, $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

(OR)

b) Prove that the medians of a triangle are concurrent.

46. a) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $\frac{1}{x^2}$ when x is large.

(OR)

b) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$.

47. a) Show that the straight lines $x^2 - 4xy + y^2 = 0$ and $x + y = 3$ form an equilateral triangle.

(OR)

b) A firm manufactures PVC pipes in three plants viz, X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production,

(i) find the probability that the selected pipe is a defective one.

(ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y ?