COMMON HALF YEARLY EXAMINATION - 2025

Standard XI

Reg.No. | | | | | | |

MATHEMATICS

Time : 3.00 hrs — Part - I — *Marks : 90*

$20 \times 1 = 20$

I. **Choose the correct answer:**

1. The number of relations on a set containing 3 elements is
   a) 9 b) 81 c) 512 d) 1024
2. The function $f : R \to R$ is defined by $f(x) = \sin x + \cos x$ is
   a) an odd function b) neither odd nor even
   c) an even function d) both odd and even
3. The value of $\log_a b \log_b c \log_c a$ is
   a) 2 b) 1 c) 3 d) 4
4. $\cos 1° + \cos 2° + \cos 3° + ... + \cos 179° =$
   a) 0 b) 1 c) –1 d) 89
5. The number of 5 digit numbers all digits of which are odd is
   a) 25 b) 5^5 c) 5^6 d) 625
6. $1 + 3 + 5 + 7 + ... + 19$ is equal to
   a) 101 b) 81 c) 71 d) 100
7. The H.M of two positive numbers whose AM and GM are 16, 8 respectively is
   a) 10 b) 6 c) 5 d) 4
8. Value of $\sin^2\theta - \cos^2\theta$ is
   a) $\sin 2\theta$ b) $-\cos 2\theta$ c) $\cos 2\theta$ d) $-\sin 2\theta$
9. The area of triangle formed by the lines $x^2 - 4y^2 = 0$ and $x = a$ is
   a) $2a^2$ b) $\frac{\sqrt{3}}{2}a^2$ c) $\frac{1}{2}a^2$ d) $\frac{2}{\sqrt{3}}a^2$
10. The length of $\perp$ from the origin to the line $\frac{x}{3} - \frac{y}{4} = 1$ is
    a) $\frac{11}{5}$ b) $\frac{5}{12}$ c) $\frac{12}{5}$ d) $\frac{-5}{12}$
11. If the points $(x, -2)$, $(5, 2)$, $(8, 8)$ are collinear, then x is equal to
    a) –3 b) $\frac{1}{3}$ c) 1 d) 3
12. If $\lambda\hat{i} + 2\lambda\hat{j} + 2\lambda\hat{k}$ is a unit vector, then the value of λ is
    a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{9}$ d) $\frac{1}{2}$
13. If $\vec{a} + 2\vec{b}$ and $3\vec{a} + m\vec{b}$ are parallel, then the value of m is
    a) 3 b) $\frac{1}{3}$ c) 6 d) $\frac{1}{6}$

14. Value of $\lim_{x\to 0} \frac{e^x-1}{x} - 1 =$

a) 0 b) 1 c) –1 d) none of these

15. If $pv = 81$, then $\frac{dp}{dv}$ at $v = 9$ is

a) 1 b) –1 c) 2 d) –2

16. If $y = mx + c$ and $f(0) = f'(0) = 1$, then $f(2)$ is

a) 1 b) 2 c) 3 d) –3

17. $\int e^{\sqrt{x}}\,dx$ is

a) $2\sqrt{x}\left(1-e^{\sqrt{x}}\right)+c$ b) $2\sqrt{x}\left(e^{\sqrt{x}}-1\right)+c$

c) $2\sqrt{x}\left(1-\sqrt{x}\right)+c$ d) $2\sqrt{x}\left(\sqrt{x}-1\right)+c$

18. If $\int f(x)dx = g(x)+c$, then $\int f(x)\,g'(x)dx =$

a) $\int (f(x))^2\,dx$ b) $\int f(x)\,g(x)\,dx$ c) $\int f'(x)\,g(x)\,dx$ d) $\int (g(x))^2\,dx$

19. Ten coins are tossed. The probability of getting at least 8 heads is

a) $\frac{7}{64}$ b) $\frac{7}{32}$ c) $\frac{7}{16}$ d) $\frac{7}{128}$

20. If two events A and B are independent such that $P(A) = 0.35$ and $P(A \cup B) = 0.6$, then $P(B)$ is

a) $\frac{5}{13}$ b) $\frac{1}{13}$ c) $\frac{4}{13}$ d) $\frac{7}{13}$

Part - II

II. **Answer any 7 questions. (Q.No.30 is compulsory)** **7 x 2 = 14**

21. Find the number of subsets of A if $A = \{x : x = 4n+1, 2 \le n \le 5, n \in N\}$

22. Find the real roots of $x^4 = 16$

23. Find the rank of the word "TABLE"

24. If θ is a parameter, find the equation of the locus of a moving point, whose co-ordinates are $x = a\cos^3\theta$ and $y = a\sin^3\theta$

25. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$

26. Find the angle between the Vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$

27. Find the positive integer n, so that $\lim_{x\to 3} \frac{x^n-3^n}{x-3} = 27$

28. Evaluate $\int e^{3x} \cos 2x \, dx$

29. What is the chance that leap year should have fifty three Sundays?

30. If $y = \tan^{-1}x$, find y''

Part - III

III. **Answer any 7 questions. (Q.No.40 is compulsory)** 7 x 3 = 21

31. In the Z of integers, define mRn if m–n is divisible by 7, prove that R is an equivalence.

32. Resolve into partial fractions : $\frac{x}{(x+3)(x-4)}$

33. Prove that $\tan(315°)\cot(-405°) + \cot(495°)\tan(-585°) = 2$

34. ${}^{n}P_{r} = 11880$ and ${}^{n}C_{r} = 495$, find n and r

35. Find $\frac{d^2y}{dx^2}$ if $x^2+y^2 = 4$

36. Show that collinear (1, 3) , (2, 1) , $\left(\frac{1}{2}, 4\right)$

37. Show that the vectors are coplanar: $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$

38. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

39. Evaluate $\int x \cos x \, dx$

40. Find $\sqrt[3]{999}$ approximately (two decimal places)

Part - IV

IV. **Answer all the questions.** 7 x 5 = 35

41. a) Let f, g : R → R be defined as f(x) = 2x– |x| and g(x) = 2x + |x| find fog

(OR)

b) Simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$.

42. a) If A+B = 45°, then prove that (1 + tan A) (1 + tan B) = 2

(OR)

b) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove that xyz = 1

43. a) By Mathematical Induction method, Prove that

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(OR)

b) State and prove Napier's Theorem for ΔABC

44. a) Evaluate $\int \frac{5x-7}{\sqrt{3x-x^2-2}}\,dx$

(OR)

b) $y = e^{\tan^{-1}x}$, show that $(1+x^2)\,y'' + (2x-1)\,y' = 0$

45. a) Using factor theorem, Prove that

$$\begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$$

(OR)

b) If $\sin y = x \sin(a+y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$, $a \neq n\pi$

46. a) Prove that the medians of triangle are concurrent

(OR)

b) A factory has two machines I and II. Machine–I produces 40% of items of the out put and Machine–II produces 60% of the items. Further 4% of items produced by Machine–I are defective and 5% produced by Machine–II are defective. If an item is drawn at random, find the probability that it is a defective item.

47. a) The slope of one of the straight lines $ax^2+2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.

(OR)

b) Prove that $\sqrt[3]{x^3+6} - \sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

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