HALF YEARLY EXAMINATION – 2025

STD : 12 **MATHEMATICS** **MARK : 90** **TIME : 3 hrs**

PART – A

CHOOSE THE CORRECT ANSWER :- **20 X 1 = 20**

1) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj}A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$

(a) 0 (b) $\sin\theta$ (c) $\cos\theta$ (d) 1

2) If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0, (\cos\theta)x - y + z = 0,$ $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

3) The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

4) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(a) $cis\frac{2\pi}{3}$ (b) $cis\frac{4\pi}{3}$ (c) $-cis\frac{2\pi}{3}$ (d) $-cis\frac{4\pi}{3}$

5) *If α, β and γ are the roots of $x^3 + px^2 + qx + r$*, then $\sum\frac{1}{\alpha}$ is

(a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

6) The polynomial $x^3 - kx^2 + 9x$ has three real roots if and only if, k satisfies

(a) $|k| \le 6$ (b) $k = 0$ (c) $|k| > 6$ (d) $|k| \ge 6$

7) $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then x is a root of the equation

(a) $x^2 - x - 6 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$

8) The value of $cos^{-1}x + cos^{-1}(-x)$ is

(a) 0 (b) π (c) $2cos^{-1}x$ (d) $cos^{-1}x$

9) The area of quadrilateral formed with foci of the hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is

(a) $4(a^2 + b^2)$ (b) $2(a^2 + b^2)$ (c) $a^2 + b^2$ (d) $\frac{1}{2}\left(a^2 + b^2\right)$

10) If the two tangents drawn from a point P to the parabola $y^2 = 4$ ▯ are at right angles then the locus of P is

(a) $2x + 1 = 0$ (b) $x = -1$ (c) $2x - 1 = 0$ (d) $x = 1$

11) The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

12) If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1, \lambda>0$ is $\frac{1}{5}$, then the value of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

13) The volume of a sphere is increasing in volume at the rate of $3\pi cm^3/sec$. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(a) 3cm/s (b) 2cm/s (c) 1cm/s (d) $\frac{1}{2}$cm/s

14) The slant asymptotes for the function $f(x)=\frac{x^2+3x+2}{x-1}$ is

(a) $y=x+4$ (b) $y=x+5$ (c) $y=x+2$ (d) $y=x+1$

15) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

16) If $u(x,y)=x^2+3xy+y-2019$, then $\left.\frac{\partial u}{\partial x}\right|_{(4,-5)}$ is equal to

(a) -4 (b) -3 (c) -7 (d) 13

17) The value of $\int_0^{2/3}\frac{dx}{\sqrt{4-9x^2}}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) π

18) If $\int_0^1 x^n(1-x)^n\,dx=\frac{1}{140}$, then the value of n is

(a) 1 (b) 2 (c) 3 (d) 4

19) The slope at any point of a curve $y=f(x)$ is given by $\frac{dy}{dx}=3x^2$ and it passes through $(-1,1)$. Then the equation of the curve is

(a) $y=x^3+2$ (b) $y=3x^2+4$ (c) $y=3x^3+4$ (d) $y=x^3+5$

20) The order and degree of the differential equation $\frac{d}{dx}\left(\frac{d^2y}{dx^2}+1\right)=x$ is

(a) 2 , 2 (b) 2 , 1 (c) 3 , 2 (d) 3 , 1

PART – B

ANSWER ANY SEVEN QUESTIONS (Q.NO : 30 IS COMPULSORY) :- **7 X 2 = 14**

21. Find the rank of the matrices $\begin{bmatrix}1 & -2 & 3\\ 2 & 4 & -6\\ 5 & 1 & -1\end{bmatrix}$

22. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

23. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.

24. Find the value of $sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

25. Obtain the equation of the circle for which (3,4) and (2,−7) are the ends of a diameter.

26. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}$ are also coplanar.

27. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the $x-$axis for the functions $f(x) = x^2 - x,\ x \in [0,1]$

28. Evaluate: $\int_0^{\frac{\pi}{2}} sin^5 x\, cos^4 x\, dx$.

29. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.

30. Find the approximate change in the volume V of a cube of side 2 meters caused by increasing the side by 2%.

PART – C

ANSWER ANY SEVEN QUESTIONS (Q.NO : 40 IS COMPULSORY) :- **7 X 3 = 21**

31. Test for consistency and if possible, solve the following systems of equations by rank method.
$$2x + 2y + z = 5,\ x - y + z = 1,\ 3x + y + 2z = 4$$

32. Show that the equation $z^2 = \bar{z}$ has four solutions.

33. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3: 2.

34. Find the domain of the function $f(x) = sin^{-1}\left(\frac{x^2+1}{2x}\right)$

35. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

36. Find the absolute extrema of the function $f(x) = 3\,cos\,x$ on the closed interval $[0,2\pi]$.

37. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.

38. Evaluate: $\int_0^1 |5x - 3|\, dx$.

39. Suppose a person deposits ₹10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

40. Find the shortest distance between the skew lines $\vec{r} = (\vec{i} - \vec{j}) + \lambda(2\vec{i} + \vec{j} + \vec{k})$ and $\vec{r} = (\vec{i} + \vec{j} - \vec{k}) + \mu(2\vec{i} - \vec{j} - \vec{k})$

PART – D

ANSWER ALL THE QUESTIONS :- 7 X 5 = 35

41 a. Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$.

(OR)

b. The region enclosed between the graphs of $y = x$ and $y = x^2$ is denoted by R, Find the volume generated when R is rotated through $360°$ about x-axis.

42 a. Find the vector parametric form and Cartesian form of the equation of the plane passing through the points $(-1, 2, 0), (2, 2, -1)$and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$.

(OR)

b. If F is the constant force generated by the motor of an automobile of mass M, its velocity v is given by $M\frac{dv}{dt} = F - kv$, where k is a constant. Express v in terms of t given that $v = 0$ when $t = 0$.

43. a. Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$.

(OR)

b. Prove by vector method that $sin(\alpha + \beta) = sin\,\alpha\, cos\,\beta + cos\,\alpha\, sin\,\beta$.

44. a. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

(OR)

b. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

45. a. If $w(x, y, z) = x^2 + y^2 + z^2, x = e^t, y = e^t\, sin\, t$ and $z = e^t\, cos\, t$, find $\frac{dw}{dt}$.

(OR)

b. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

46. a. If $cos^{-1} x + cos^{-1} y + cos^{-1} z = \pi$ and $0 < x, y, z < 1$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$.

(OR)

b. Solve : $(1 + x^3)\frac{dy}{dx} + 6x^2y = 1 + x^2$.

47. a. Solve the following system of linear equations

$2x - y + z = 7$, $3x + y - 5z = 13$, $x + y + z = 5$ by matrix inversion method.

(OR)

b. If $z = x + iy$ and $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$, show that $\sqrt{3}\,x^2 + \sqrt{3}\,y^2 - 2y - \sqrt{3} = 0$.