Tsi12M

Tenkasi District
Common Half Yearly Examination - 2025

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Standard 12
MATHEMATICS

Time: 3.00 Hours | Marks: 90

I. Answer all the questions. **20x1=20**

1) If $A = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

a) $-4/5$ b) $-3/5$ c) $3/5$ d) $4/5$

2) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then adj (adj A) is

a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ c) $\begin{matrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{matrix}$ d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

3) If $|z - 2 + i| \leq 2$ then the greatest value of $|z|$ is

a) $\sqrt{3} - 2$ b) $\sqrt{3} + 2$ c) $\sqrt{5} - 2$ d) $\sqrt{5} + 2$

4) The principle argument of $(\sin 40^\circ + i \cos 40^\circ)^5$ is

a) -110° b) -70° c) 70° d) 110°

5) If α, β, γ are the zeros of x^3+px^2+qx+r then $\frac{1}{\alpha}$ is

a) $-q/r$ b) $-p/r$ c) q/r d) $-q/p$

6) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) [1, 2] b) [-1, 1] c) [0, 1] d) [-1, 0]

7) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) $\frac{a}{b}$

8) The semi transverse axis and semi conjugate axis length of the hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$ is

a) 4, 5 b) 5, 4 c) 8, 10 d) 10, 8

9) The volume of the parallelepiped with its edges represented by the vectors $\bar{i} + \bar{j}, \bar{i} + 2\bar{j}, \bar{i} + \bar{j} + \pi\bar{k}$

a) $\pi/2$ b) π c) $\pi/4$ d) $\pi/3$

10) The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}$, z=2 and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

11) The minimum value of the fun. 13–x1+9 is

a) 0 b) 3 c) 6 d) 9

12) The point of inflexion of the Curve $y=(x-1)^3$ is

a) (0,0) b) (0, 1) c) (1, 0) d) (1, 1)

13) If $f(x,y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$

14) The value of $\int_0^\infty e^{-3x}\, 3x^2 dx$ is

a) $\frac{7}{27}$ b) $\frac{5}{27}$ c) $\frac{4}{27}$ d) $\frac{2}{27}$

15) The area of the curve xy = 12 bounded by y=0, x = 1 and $x = e^2$

a) 6 b) 12 c) 18 d) 24

16) The solution of $\frac{dy}{dx} + p(x)y = 0$ is

a) $y = ce^{\int pdx}$ b) $y = ce^{-\int pdx}$ c) $x = ce^{-\int pdy}$ d) $x = ce^{\int pdy}$

17) The number of arbitrary constants in the general solution of order n and n+1 respectively

a) n - 1, n b) n, n+1 c) n+1, n+2 d) n+1, n

18) If P(x=0)=1-p(x=1). If E(x) = 3 var (x) then p(x=0) is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

19) Let x have a Bernoulli distribution with mean 0.4 then the variance of (2x-3) is

a) 0.24 b) 0.48 c) 0.6 d) 0.96

20) Which one of the following is incorrect? for any two propositions p and q, we have

a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

c) $\neg(p \vee q) \equiv \neg p \vee \neg q$ d) $\neg(\neg p) \equiv p$

II. Answer any 7 questions. Q.No. 30 is compulsory 7x2=14

21) If $\text{adjA} = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}

22) If |z| = 3, S.T $7 \le |z + 6 - 8i| \le 13$

23) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

24) Find the equation of the parabola with focus $\left(-\sqrt{2}, 0\right)$ and directrix $x = \sqrt{2}$

25) A particle is acted upon by the forces $3\bar{i} - 2\bar{j} + 2\bar{k}$ and $2\bar{i} + \bar{j} - \bar{k}$ is displaced from the point (1, 3, -1) to line point (4, -1, λ). If the workdone by the forces is 16 units, find the value of λ

26) Prove that the fun. $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$

27) Find df for $f(x) = x^2 + 3x$ and evaluate it for x= 2 and dx = 0.1

28) Evaluate $\int_0^1 x^3(1-x)^4 dx$

29) Let x be a random variable denoting the life time of an electrical equipment having density fun. $f(x) = \begin{cases} ke^{-2x}, & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$ find the value of k.

30) Solve : $\frac{dy}{dx} = e^{2x-y} + x^3 e^{-y}$

III. Answer any 7 questions. Q.No. 40 is compulsory **7x3=21**

31) Show that $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

32) Solve the equation $9x^3-36x^2+44x-16 = 0$ if the roots form an Arithmetic Progression.

33) If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2' then prove that $t_2 = -t_1+\frac{2}{t_1}$

34) If the straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{m^2}$ and $\frac{x-3}{1}=\frac{y-2}{m^2}=\frac{z-1}{2}$ are coplanar, find the distinct real values of m.

35) Expand tan x in ascending powers of x upto 5th power for $-\pi/2 < x < \pi/2$

36) Let $f(x) = \sqrt[3]{x}$ find the linear approximation at x = 27. Use the linear approximation to approximate $\sqrt[3]{27.2}$

37) If $\int_0^\infty e^{-\alpha x^2}x^3\,dx = 32\ \alpha > 0$ find α

38) Solve: $\frac{dy}{dx}+2y\cot x = 3x^2\csc^2 x$

39) Showthat $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

40) Solve by Cramer's Rule: $\frac{2}{x}+\frac{3}{y}+\frac{1}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{3}{z}=-7,\ \frac{3}{x}-\frac{5}{y}+\frac{2}{z}=-5$

IV. Answer all the questions. **7x5=35**

41) Find the value of K for which the equation $Kx-2y+z = 1$, $x-2Ky+z = -2$, $x-2y+Kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution

(OR)

Find mean and variance of a raandom variable x whose prob. density fun is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

42) If $z = x+iy$ and $\arg\frac{z-i}{z+2}=\frac{\pi}{4}$ show that $x^2+y^2+3x-3y+2 = 0$

(OR)

Evaluate: $\int_{\pi/8}^{3\pi/8}\frac{1}{1+\sqrt{\cot x}}dx$

43) Solve the equation $6x^4-5x^3-38x^2-5x+6 = 0$ if it is known that $1/3$ is a solution.

(OR)

If $U = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$. Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{1}{2}\cot u = 0$ by uing Euler's Theorem.

44) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z = \pi$ and $0 < x, y, z < 1$ showthat $x^2+y^2+z^2+2xyz = 1$.

(OR)

A hollow cone with base radius a cm and height b cm is placed on a table. Showthat the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

45) Idenfity the type of conic and find centre, foci, vertices, and directrices of $18x^2+12y^2-144x+48y+120 = 0$

(OR)

Verify (i) closure property (ii) commutative property (iii) Associative property (iv) existence of identity and (v) existence of inverse for the operation X_{11} on a subset A = {1, 3, 4, 5, 9} of the set of remainders {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

46) Using vector method provethat $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

(OR)

Using integration find the area of the region bounded by the ΔABC. whose vertices are A(−1, 1) B(3, 2) and C(0, 5) respetively.

47) Find non-parametric form of vector equation and Cartesian equation of the plane passing through the points (−1, 1, 1) and (1, −1, 1) and $\perp^r$ to the plane x+2y+2z = 5.

(OR)

Water at temperature 100°C cools in 10 minutes to 80°C in the room temperature of 25°C find (i) the temperature of water after 20 minutes (ii) the time when the temperature is 40°C $\log_e \frac{11}{15} = -0.3101, \log_e 5 = 1.6094$

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