V12M

Virudhunagar District

Common Halfy Yearly Examination, - 2025

Standard 12

MATHEMATICS

Time: 3.00 Hours Marks: 90

Part - I

Choose the best answer: **20x1=20**

1) If $A = \begin{matrix} 3/5 & 4/5 \\ x & 3/5 \end{matrix}$ and $A^T = A^{-1}$, then the value of x is

a) $-4/5$ b) $-3/5$ c) $3/5$ d) $4/5$

2) If A is an invertible matrix of order 2 then det (A^{-1}) is equal to

a) det (A) b) $\frac{1}{\det(A)}$ c) 1 d) 0

3) The conjugate of a complex number is $\frac{1}{i-2}$. Then the complex number is

a) $\frac{1}{i+2}$ b) $\frac{-1}{i+2}$ c) $\frac{-1}{i-2}$ d) $\frac{1}{i-2}$

4) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

a) cis $2\pi/3$ b) cis $4\pi/3$ c) $-$cis $2\pi/3$ d) $-$cis $4\pi/3$

5) The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^{n}C_r (-1)^r x^r$ is

a) 0 b) n c) < n d) r

6) $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$

a) $\pi/2$ b) $\pi/3$ c) $\pi/4$ d) $\pi/6$

7) The eccentricity of the ellipse $(x-3)^2+(y-4)^2 = y^2/9$ is

a) $\sqrt{3}/2$ b) $1/3$ c) $1/3\sqrt{2}$ d) $1/\sqrt{3}$

8) If y = mx+c is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

a) $c^2 = a^2m^2+b^2$ b) $b^2 = c^2+a^2m^2$ c) $c^2=a^2m^2+m^2$ d) $c^2=a^2m^2-b^2$

9) Distance from the origin to the plane $3x-6y+2z+7 = 0$ is

a) 0 b) 1 c) 2 d) 3

10) If $a = i + j + k$, $b = i + j$, $c = i$ and $(a \times b) \times c = \lambda a + \mu b$, then the value of $\lambda+\mu$ is

a) 0 b) 1 c) 6 d) 3

11) The point of inflection of the curve $y = (x-1)^3$ is
a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)

12) The vertical asymptote of $f(x) = \frac{x^2}{x+1}$ is
a) $x = -1$ b) $x = 1$ c) $y = 1$ d) $y = -1$

13) Linear approximation for $g(x) = \cos x$ at $x = \pi/2$ is
a) $x + \pi/2$ b) $-x + \pi/2$ c) $x - \pi/2$ d) $-x - \pi/2$

14) The value $\int_0^{\pi/6} \cos^3 3x\,dx$ is
a) $2/3$ b) $2/9$ c) $1/9$ d) $1/3$

15) $\int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx =$
a) a b) 0 c) $a/2$ d) 2a

16) The order and degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 6y = 5\cos 3x$ is
a) 4, 7 b) 4, 3 c) 3, 7 d) 1, 7

17) If $f(x) = \begin{cases} 2x & , 0 \le x \le a \\ 0 & , \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is
a) 1 b) 2 c) 3 d) 4

18) If in 6 trials, x is a binomial variable which follows the relation $9P(x = 4) = P(x = 2)$ then the probability of success is
a) 0.125 b) 0.25 c) 0.375 d) 0.75

19) Subtraction is not a binary operation in
a) R b) Z c) N d) Q

20) Which one is the contrapositive of the statement $(p \vee q) \to r$?
a) $\neg r \to (\neg p \wedge \neg q)$ b) $\neg r \to (p \vee q)$
c) $r \to (p \wedge q)$ d) $p \to (q \vee r)$

Part - II

Answer any 7 questions: Question no. 30 is compulsory: **7x2=14**

21) Find the rank of the matrix $\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

22) Write in rectangular form $\frac{10 - 5i}{6 + 2i}$

23) Find the principal value of $\sin^{-1}\ \sin\ \frac{5\pi}{6}$

24) Obtain the equation of the circle for which (3, 4) and (2, −7) are the ends of the diameter.

25) Evaluate: $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

26) If $f(x, y) = x^3-3x^2+y^2+5x+6$, then find f_x at (1, −2)

27) Evaluate: $\int_0^{\infty} x^5 e^{-3x}\, dx$

28) Solve: $\frac{dy}{dx} + 2y = e^{-x}$

29) $\neg p \vee q$ construct the truth table.

30) Find the acute angle between the planes $\vec{r}.(2\hat{i} + 2\hat{j} + 2\hat{k}) = 11$ and $4x-2y+2z = 15$

Part - III

Answer any 7 questions: Question no. 40 is compulsory: **7x3=21**

31) Solve by Cramer's rule: $x+y-z = 3$; $3x-2y+3z = 5$; $2x-3y+4z = 1$

32) If $|z| = 2$, show that $3 \le |z+3+4i| \le 7$

33) Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.

34) A concrete bridge is designed as a parabolic arch. The road over bridge is 40 m long and the maximum height of the arch is 15 m. Write an equation of the parabolic arch.

35) Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$

36) Find the equation of the tangent and normal to the curve $y = x^2+3x-2$ at the point (1, 2)

37) Evaluate: $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}}\, dx$

38) Solve: $\frac{dy}{dx} = (3x+y+4)^2$

39) If X is the randam variable with distribution function $f(x) = \begin{cases} 0 & , x < 0 \\ x & , 0 \le x < 1 \\ 1 & , 1 \le x \end{cases}$ then find (i) probability density function f(x) (ii) $P(0.2 \le X \le 0.7)$

40) Construct the truth table for $(\neg p \to r) \wedge (p \leftrightarrow q)$

Part - IV

Answer all the questions: **7x5=35**

41) a] Solve the following system of linear equations by Gaussian elimination method. $x-y+2z = 2,\ 2x+y+4z = 7,\ 4x-y+z = 4$

(OR)

b] A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic meter per minutes, how fast is the depth of the water increases when the water is 8 meters deep?

42) a] Solve the equation $z^3+8i = 0$ where $z \in c$.

(OR)

b] Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

43) a] It is known that the roots of the equation $x^3-6x^2-4x+24 = 0$ are in arithmetic progression. Find its roots.

(OR)

b] Solve: $(1+x^3)\frac{dy}{dx}+6x^2y = 1+x^2$

44) a] Solve: $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

(OR)

b] The probability density function of x is given by $f(x) = \begin{cases} K & ,1 \le x \le 5 \\ 0 & ,\text{otherwise} \end{cases}$

find (i) distribution function (ii) $P(X < 3)$ (iii) $P(2 < x < 4)$ (iv) $P(3 \le X)$

45) a] Find the equation of the circle passing through the points (1, 0) (−1, 0) and (0, 1)

(OR)

b] Find the vector and Cartesian equations of the plane containing the line $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ and passing through the point (−1, 1, −1)

46) a] Prove that among all the rectangles of the given perimeter, the square has the maximum area.

(OR)

b] A rod of length 1.2 m moves with its ends always touching the co-ordinate axes the locus of point P on the rod, which is 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.

47) a] Let $g(x, y) = 2y+x^2$, $x = 2r-s$, $y = r^2+2s$, $r, s \in R$ find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

(OR)

b] Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identify and (v) existence of inverse for the operation $+_6$ on Z_6 using table corresponding to addition modulo 6.

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