

COMMON QUARTERLY EXAMINATION-2023-24

Time Allowed : 3.00 Hours]

MATHEMATICS

[Max. Marks : 100

SECTION - I

I. Answer all of the following:

14x1=14

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is -----
 a) 1 b) 2 c) 3 d) 6
2. If $\{(a, 8), (6, b)\}$ represents an identity function then the value of a and b are respectively
 a) (8, 6) b) (8, 8) c) (6, 8) d) (6, 6)
3. $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is -----
 a) Linear b) Cubic c) Reciprocal d) Quadratic
4. If the HCF of 65 and 117 is expressible in the form of $65m - 117$. Then the value of m is -----
 a) 4 b) 2 c) 1 d) 3
5. $7^{4k} \equiv \text{-----} \pmod{100}$
 a) 1 b) 2 c) 3 d) 4
6. If $1 + 2 + 3 + \dots + n = k$ then $1^3 + 2^3 + 3^3 + \dots + n^3 = \text{-----}$
 a) K^3 b) K^2 c) $\frac{K(K+1)}{2}$ d) $(k+1)^3$
7. $y^2 + \frac{1}{y^2}$ is not equal to
 a) $\frac{y^4 + 1}{y^2}$ b) $\left(y + \frac{1}{y}\right)^2$ c) $\left(y - \frac{1}{y}\right)^2 + 2$ d) $\left(y + \frac{1}{y}\right)^2 - 2$
8. The solution of $(2x - 1)^2 = 9$ is equal to
 a) -1 b) 2 c) -1, 2 d) None of these
9. If $\triangle ABC$ is an isosceles triangle with $\angle C = 90^\circ$ and $AC = 5$ cm, then AB is
 a) 2.5 cm b) 5 cm c) 10 cm d) $5\sqrt{2}$ cm
10. If in $\triangle ABC$, $DE \parallel BC$, $AB = 3.6$ cm, $AC = 2.4$ cm and $AD = 2.1$ cm then the length of AE is
 a) 1.4 cm b) 1.8 cm c) 1.2 cm d) 1.05 cm
11. The straight line given by the equation $x = 11$ is
 a) Parallel to X axis b) Parallel to Y axis
 c) Passing through the origin d) Passing through the point (0, 11)
12. Slope of line $ax + by + c = 0$ is -----
 a) $\frac{b}{a}$ b) $\frac{a}{b}$ c) $-\frac{b}{a}$ d) $-\frac{a}{b}$
13. (2, 1) is the point of intersection of two lines
 a) $x - y - 3 = 0$; $3x - y - 7 = 0$ b) $x + y = 3$; $3x + y = 7$
 c) $3x + y = 3$; $x + y = 7$ d) $x + 3y - 3 = 0$; $x - y - 7 = 0$.
14. $\tan\theta \operatorname{cosec}^2\theta - \tan\theta$ equal to
 a) $\sec\theta$ b) $\cot^2\theta$ c) $\sin\theta$ d) $\cot\theta$.

SECTION - II

II. Answer any 10 questions. Question No. 28 is compulsory.

10x2=20

15. A Relation R is given by the set $\{(x, y) / y = x+3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range.
16. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = [f(x)]^2$.
17. Let f be a function $f: N \rightarrow N$ be defined by $f(x) = 3x + 2, x \in N$ (i) Find the images of 1 and 2
 ii) Find the pre-images of 29 and 53.
18. If $13824 = 2^a \times 3^b$ then find a and b .
19. Find a_n and a_{16} whose n^{th} term is

$$a_n = \begin{cases} \frac{n^2-1}{n+3} & ; n \text{ is even, } n \in N \\ \frac{n^2}{2n+1} & ; n \text{ is odd, } n \in N \end{cases}$$

20. Find the sum $3 + 1 + \frac{1}{3} + \dots \infty$.
21. Find the excluded values of $\frac{x^3 - 27}{x^3 + x^2 - 6x}$.
22. Determine the quadratic equation whose sum and product of -9 and 20.
23. If $\triangle ABC$ is similar to $\triangle DEF$ such that $BC = 3$ cm, $EF = 4$ cm and area of $\triangle ABC = 54$ cm². Find the area of $\triangle DEF$.
24. In $\triangle ABC$, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15$ cm find AE.
25. The line through the points $(-2, a)$ and $(9, 3)$ has slope $-\frac{1}{2}$. Find the value of a.
26. Find the intercepts made by the line $4x - 9y + 36 = 0$ on the co-ordinate axis.
27. Prove that $\sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}} = \operatorname{cosec}\theta + \cot\theta$
28. If the points $(p^2, 0)$, $(0, q^2)$ and $(1, 1)$ are straight line. Then prove that $\frac{1}{p^2} + \frac{1}{q^2} = 1$

SECTION - III

III. Answer the following any 10 questions. Q.No.42 is compulsory.

10x5=50

29. Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
30. If the function $f: R \rightarrow R$ is defined by $f(x) = \begin{cases} 2x + 7; & x < -2 \\ x^2 - 2; & -2 \leq x < 3 \\ 3x - 2; & x \geq 3 \end{cases}$ then find the values of
- i) $f(4)$ ii) $f(-2)$ iii) $f(4) + 2f(1)$ iv) $\frac{f(1) - 3f(4)}{f(-3)}$
31. If $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$. Prove that $(f \circ g) \circ h = f \circ (g \circ h)$.
32. In an A.P, sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers
33. Find the sum to n terms of $3 + 33 + 333 + \dots$
34. Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm.....24cm. How much area can be decorated with these colour papers?
35. Solve : $x + y + z = 5$; $2x - y + z = 9$; $x - 2y + 3z = 16$.
36. If $36x^4 - 60x^3 + 61x^2 - mx + n$ is a perfect square, find the values of m and n.
37. Prove that the equation $x^2(p^2 + q^2) + 2x(pr + qs) + r^2 + s^2 = 0$ has no real roots. If $ps = qr$ then show that the roots are real and equal.
38. State and prove Angle bisector theorem.
39. Find the area of the quadrilateral formed by the points $(8, 6)$, $(5, 11)$, $(-5, 12)$ and $(-4, 3)$.
40. Find the equation of perpendicular bisector of line joining the points A $(-4, 2)$ and B $(6, -4)$.
41. If $\frac{\cos\theta}{1 + \sin\theta} = \frac{1}{a}$ then prove that $\frac{a^2 - 1}{a^2 + 1} = \sin\theta$
42. The sum of the reciprocals of $(x+2)$ and $(x-2)$ is equal to 6 times of the reciprocal of $4x+7$. Find the value of x.

SECTION - IV

IV. Answer the following.

2x8=16

43. a) A bus is travelling at a uniform speed of 50 km/hr. Draw the distance - time graph and hence find
i) the constant of variations ii) how far will it travel in 90 minutes? iii) the time required to cover a distance of 300 km from the graph.

(OR)

- b) Draw the graph of $xy = 24$, $x, y > 0$ using the graph find (i) y when $x = 3$ and (ii) x when $y = 6$.
43. a) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the $\triangle PQR$ (scale factor $\frac{7}{3} < 1$)

(OR)

- b) Construct a $\triangle PQR$ such that $QR = 6.5$ cm, $\angle P = 60^\circ$ and the altitude from P to QR is of length 4.5 cm